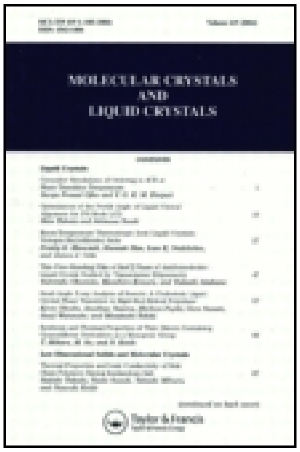

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Influence on the Hysteresis Effect of Various Parameters in Supertwisted Nematic Liquid Crystals

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Influence on the Hysteresis Effect of Various Parameters in Supertwisted Nematic Liquid Crystals

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The influence of materials and cell parameters on the hysteresis effect in a supertwisted nematic LCD composed of chiral nematic layers is theoretically investigated.

The variation of tilt angle of the director in a midplane of the LC layer with an applied voltage change is numerically calculated.

Transition voltage V_{n1}, recovery voltage V_{n2} and reduced hysteresis width $H = (V_{n1} - V_{n2})/V_{n1}$ are evaluated under the different combinations of such parameters as the elastic constant ratios $k = k_{33}/k_{11}$ and $\xi = k_{33}/k_{22}$, dielectric anisotropy $\Delta\epsilon$, pretilt angles θ_o on the electrode surfaces, total twist angle Φ and the thickness of the LC layer d.

The lower limits of the cell thickness and the pretilt angle, as well as the total twist angle, below which the bistable range completely disappears, are found to exist. The influence of various parameters on those characters are investigated.

Keywords: SBE type LCD, supertwisted LCD, bistable switching of LCD

I. INTRODUCTION

Recently, the demand for LCDs having large information capacities has been growing enormously, particularly for use with portable computers. The variety of twisted nematic (TN) devices has been widely used in portable computers. However, the limitation of the contrast ratio, the narrow viewing angle and relatively poor brightness etc. of those devices are left to be solved as problems.

The supertwisted birefringence effect (SBE) display[1-3] device as well as the surface stabilised ferroelectric liquid crystal (SSFLC) device[4,5] are currently regarded as the most prominent alternatives in LCD technology. The name SBE display, which was proposed by T. J. Scheffer and J. Nehring at first, comes from the fact that the layer twist angle is about three times larger in this display than in a usual TN display and that the contrast results from the interference of two optical normal modes rather than from the guiding of a single mode. The fundamental difference in the cell construction between a SBE display and a conventional TN display lies in that the former has a great (~270°) twist angle, a high pretilt angle and nonconventional orientation of the polarizers.

In such a cell, an abrupt transition between two bistable states in molecular orientation is induced when a voltage higher than a certain value is applied, resulting in a steep change of the optical transmittance. For those bistable states, the total twist angles are the same, while the center-layer tilt angles are different from each other.

Generally, the threshold voltage V_{n1}, at which the transition from the lower energy state to the higher one takes place, is higher than V_{n2}, at which the reverse transition takes place.

In this report, theoretical investigations are made on the influence of material parameters and cell parameters on the transition behavior between the bistable states in an SBE display, particularly on the transition voltages V_{n1} and V_{n2} and on the reduced hysteresis width defined in terms of the transition voltages.

II. ANALYTICAL CONSIDERATION

Consider a chiral nematic layer of thickness d confined between the planes $z = -d/2$ and $z = d/2$ of a Cartesian coordinate system (x, y, z) as shown in Figure 1. The twist angle and the tilt angle of the director vector $\hat{n}$ are represented by ϕ and θ, respectively, and are assumed to be dependent only on z. As the boundary conditions, it is assumed that $\theta = \theta_o$ at $z = \pm d/2$ and that $\phi = -\Phi/2$ and $\Phi/2$ at $z = -d/2$ and $z = d/2$, respectively. From Oseen-Frank's continuum theory,[6] the free energy per unit wall area is given by

$$W = \int_{-d/2}^{d/2} \left\{ \frac{1}{2} f \cdot \left(\frac{\partial \theta}{\partial z}\right)^2 + \frac{1}{2} g \cdot \left(\frac{\partial \phi}{\partial z}\right)^2 + \frac{1}{2} e \cdot \left(\frac{\partial \phi}{\partial z}\right) + \frac{1}{2} h \cdot D_z^2 \right\} dz \tag{1}$$

where f, g, h and e are defined as follows, respectively.

$$\left.\begin{aligned} f &= k_{11}\{1 + (k - 1)\sin^2\theta\} \\ g &= k_{22}\cos^2\theta\{1 + (\xi - 1)\sin^2\theta\} \\ h &= \frac{1}{\epsilon_o\epsilon_n(1 + \epsilon\sin^2\theta)} \\ e &= -2k_{22}t_o\cos^2\theta \end{aligned}\right\} \quad (2)$$

where

$$\left.\begin{aligned} k &= k_{33}/k_{11} \\ \xi &= k_{33}/k_{22} \\ \epsilon &= (\epsilon_p - \epsilon_n)/\epsilon_n \end{aligned}\right\} \quad (3)$$

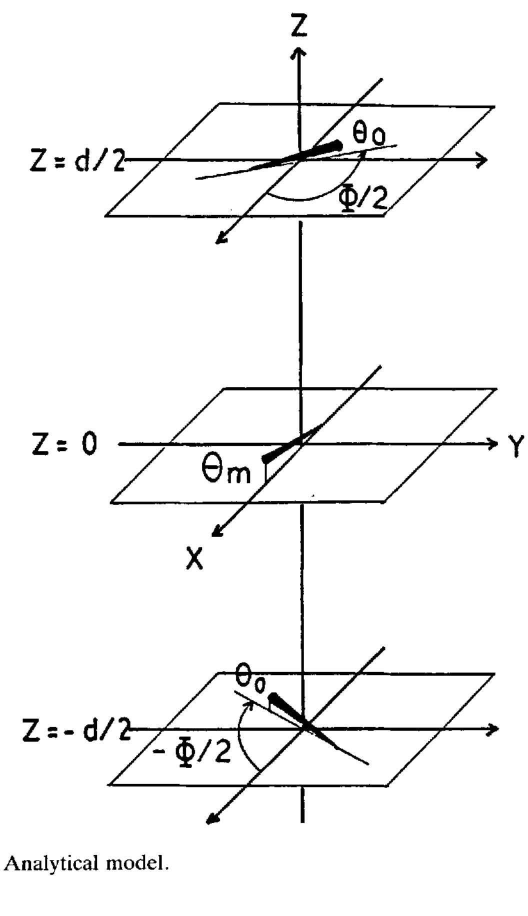

FIGURE 1 Analytical model.

The k_{11}, k_{22} and k_{33} are the liquid crystal elastic constants for splay, twist and bend, respectively, and t_o is the natural helicity of the cholesteric layer in the planar texture when boundary conditions do not cause strain in the liquid crystal, hence if the intrinsic helical pitch of a chiral nematic layer is indicated by P_o, the relation of $t_o = 2\pi/P_o$ is satisfied. The values of ϵ_p and ϵ_n are the dielectric tensor components parallel and normal to the director, respectively, ϵ_o is the vacuum dielectric constant, and D_z is the z-component of the displacement vector.

From Euler-Lagrange equations, one obtains the following differential equations.

$$2f\cdot\frac{\partial^2\theta}{\partial z^2} + f^{(1)}\cdot\left(\frac{\partial\theta}{\partial z}\right)^2 - g^{(1)}\cdot\left(\frac{\partial\phi}{\partial z}\right)^2 - e^{(1)}\cdot\left(\frac{\partial\phi}{\partial z}\right) - h^{(1)}\cdot D_z^2 = 0 \tag{4}$$

and

$$g^{(1)}\cdot\left(\frac{\partial\theta}{\partial z}\right)\left(\frac{\partial\phi}{\partial z}\right) + g\cdot\frac{\partial^2\phi}{\partial z^2} + \frac{1}{2}e^{(1)}\cdot\frac{\partial\theta}{\partial z} = 0 \tag{5}$$

where $f^{(1)}$, $g^{(1)}$, $e^{(1)}$ and $h^{(1)}$ mean the first order derivatives of f, g, e and h with respect to θ, respectively.

Integrating Eq. (5), one obtains

$$g\cdot\left(\frac{\partial\phi}{\partial z}\right) + \frac{1}{2}e = C \tag{6}$$

where C is a constant of integration.

The electric potential difference V is given by

$$V = D_z\int_{-d/2}^{d/2} h\cdot dz \tag{7}$$

Taking into account that the tilt angle θ may take an extreme value of θ_m at $z = 0$, one obtains the following relation from Eq. (4).

$$2f_m\cdot\eta_m - g_m^{(1)}\cdot\zeta_m^2 - e_m^{(1)}\zeta_m - h_m^{(1)}\cdot D_z^2 = 0 \tag{8}$$

where f_m, $g_m^{(1)}$, $e_m^{(1)}$, and $h_m^{(1)}$ are the values of f, $g^{(1)}$, $e^{(1)}$, and $h^{(1)}$ at $\theta = \theta_m$, respectively, $\eta_m = (\partial^2\theta/\partial z^2)_{z=0}$ and $\zeta_m = (\partial\phi/\partial z)_{z=0}$.

Now, the following functions are assumed as a solution of the simultaneous differential equations (4) and (5).

$$\theta = (\theta_m - \theta_o)\cos\left(\frac{\pi}{d}z\right) + \theta_o \tag{9}$$

and

$$z = a\phi^3 + b\phi \tag{10}$$

where a and b are the quantities which should be determined so as to satisfy Eq. (4) and Eq. (5).

Since $\phi = -\Phi/2$ and $\phi = \Phi/2$ at $z = -d/2$ and $z = d/2$, respectively, a is obtained from Eq. (10).

$$a = \left(\frac{2}{\Phi}\right)^3\left\{\frac{d}{2} - b\left(\frac{\Phi}{2}\right)\right\} \tag{11}$$

From Eqs. (6), and (10), one obtains

$$g_o\left\{3a\left(\frac{\Phi}{2}\right)^2 + b\right\}^{-1} + \frac{1}{2}e_o = g_m b^{-1} + \frac{1}{2}e_m \tag{12}$$

where g_o and e_o are the values of g and e for $\theta = \theta_o$, respectively. Substituting Eq. (11) into Eq. (12), one obtains

$$-(e_o - e_m)b^2 + \left\{(g_o - g_m) + \frac{3}{2}\left(\frac{d}{\Phi}\right)(e_o - e_m) + 3g_m\right\}b - 3\left(\frac{d}{\Phi}\right)g_m = 0 \tag{13}$$

The value of b is obtained from Eq. (13). Then, ζ_m is given by

$$\zeta_m = \frac{1}{b} \tag{14}$$

From Eq. (9), one obtains

$$\eta_m = -\left(\frac{\pi}{d}\right)^2(\theta_m - \theta_o) \tag{15}$$

Since $\theta(z)$ has been assumed to be the form of Eq. (9), the mean tilt angle between the electrodes is given by

$$\bar{\theta} = \frac{2}{\pi}(\theta_m - \theta_o) + \theta_o \tag{16}$$

Using the mean tilt angle $\bar{\theta}$ in Eq. (7), one may approximate Eq. (7) by

$$V = d \cdot h(\bar{\theta}) \cdot D_z \tag{17}$$

Substituting Eqs. (15) and (17) into Eq. (8) and defining the normalized voltage V_n as $V_n = V/\pi(k_{11}/\epsilon_o\Delta\epsilon)^{1/2}$, one obtains

$$V_n = (\sin 2\theta_m)^{-1/2}\frac{H_{av}}{H_m} \cdot \left[(\theta_m - \theta_o)\cdot F_m + \frac{k}{\xi}\left(\frac{d}{\pi}\right)^2 (G_m^{(1)}\zeta_m^2 + E_m^{(1)}\zeta_m)\right]^{1/2} \tag{18}$$

where $H_m = \epsilon_o\epsilon_n h_m$, $H_{av} = \epsilon_o\epsilon_n h(\bar{\theta})$, $F_m = f_m/k_{11}$, $G_m^{(1)} = g_m^{(1)}/k_{22}$, and $E_m^{(1)} = e_m^{(1)}/k_{22}$.

Considering Eq. (18) with Eqns. (2), (13) and (14), we can find that V_n depends on d and only in terms of d/P_o and $\Phi/2\pi$.

Figure 2 illustrates the calculated curves for the voltage dependence of the tilt angle of the director in a midplane of the chiral nematic layer with 28° pretilt angle at both boundaries. As Φ is decreased, the bistable range (region of negative slope) also decreases until it completely disappears. As was mentioned by T. J. Scheffer and J. Nehring, optimum multiplexing is achieved at a somewhat larger twist angle where a narrow bistable range is present. Recently, Matsumoto et al.[5] proposed a new parameter H, called the reduced hysteresis width, to estimate the multiplexibility of an SBE display, in such a way as

$$H = (V_{\text{up}}^{50} - V_{\text{down}}^{50})/V_{\text{down}}^{50} \tag{19}$$

where, V_{up}^{50} and V_{down}^{50} represent the voltages at which the optical transmittance of an SBE display becomes 50% when an applied voltage is increased and decreased, respectively. Those voltages V_{up}^{50} and

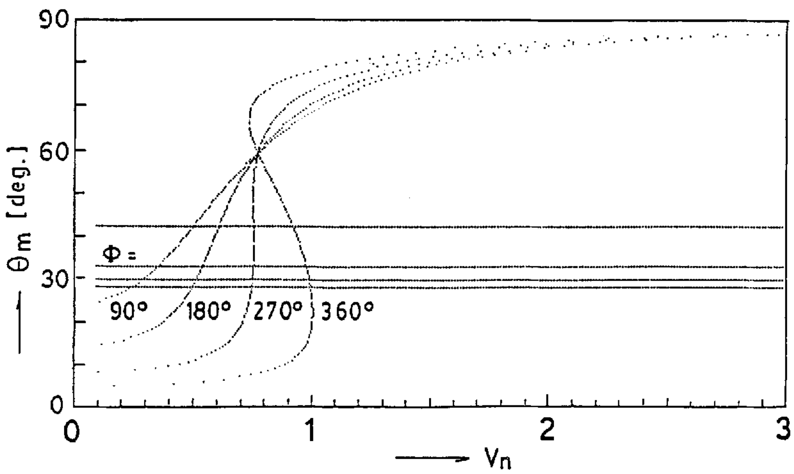

FIGURE 2 Variation of center tilt angle θ_m with normalized voltage V_n for total twist angle Φ as a parameter.

V^{50}_{down} may correspond to the voltages V_{n1} and V_{n2}, as illustrated in Figure 3. Hence, one may evaluate the parameter H by

$$H = (V_{n1} - V_{n2})/V_{n2} \tag{20}$$

It should be noted that the multiplexibility of an SBE display increases as the value of H is decreased.

III. RESULTS AND DISCUSSIONS

Except for a Section 3.5, a total twist angle of 270° is assumed.

3.1. Influence of elastic constant ratios, k and ξ

The variations of θ_m vs V_n with ξ and k calculated from Eq. (18) are shown in Figure 4. In this figure, (a) and (b) are cases of $k = 1.0$ and $k = 1.5$, respectively. In both cases, θ_o, ϵ_p and ϵ_n are fixed to 28°, 9.9 and 3.6, respectively. The cell thickness d is assumed to be $(3/4)P_o$. From these results, it is found that when ξ is decreased, the bistable range (region of negative slope) also decreases. From a comparison between (a) and (b), the critical value of ξ, below which the bistable range completely disappears, is found to be increased with an increase of k.

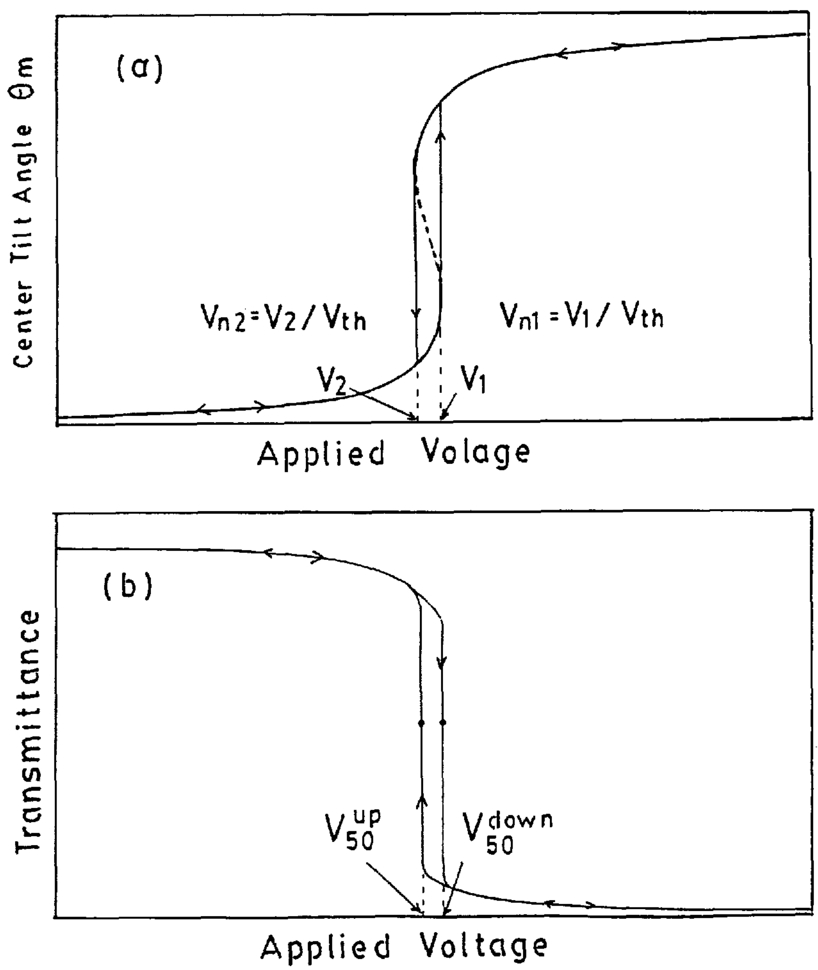

FIGURE 3 Definitions of transition voltages V_{n1} and V_{n2}.

To illustrate this tendency in more detail, transition voltages V_{n1} and V_{n2} and a reduced hysteresis width H are plotted in Figure 5 as a function of ξ. The curve parameter is k. When ξ is increased, V_{n2} decreases significantly, while V_{n1} decreases only slightly. Consequently, a reduced hysteresis width H decreases with a decrease of ξ.

From these results we can predict that a small value of k is favorable to achieve a low voltage operation, and that a high multiplexing operation may be achieved when ξ is decreased within a range in which the bistable range appears.

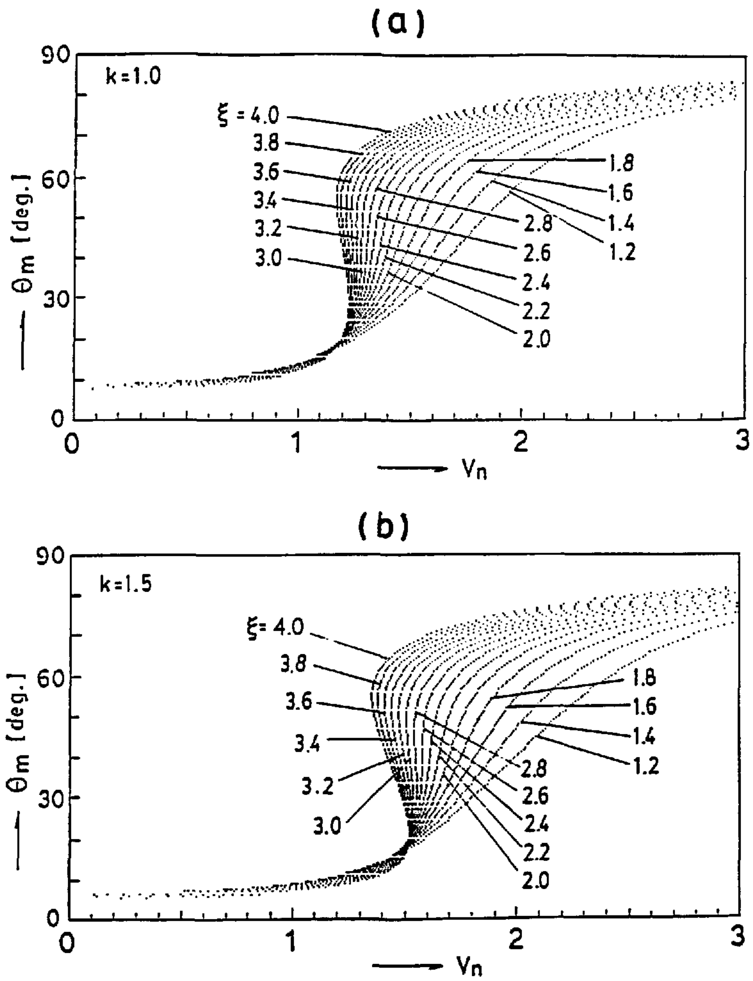

FIGURE 4 θ_m vs V_n with ξ as a parameter. (a) $k = 1.0$, (b) $k = 1.5$: $\theta_o = 28°$, $\epsilon_n = 3.6$, $\epsilon_p = 9.9$ and $d = (3/4)P_o$.

3.2. Influence of dielectric anisotropy

For the purpose of the present section, the normalized voltage V_n' is defined as $V_n' = V/\pi(k_{11}/\epsilon_o\epsilon_n)^{1/2}$ instead of V_n in Eq. (18). Apparently, V_n and V_n' satisfy the relation $V_n' = V_n/\epsilon^{1/2}$.

The calculated results for V_{n1}', V_{n2}' and H are shown in Figure 6 as a function of the dielectric constant ratio ϵ_p/ϵ_n. The curve parameters are k and ξ. From these results, both V_{n1}' and V_{n2}' are found to decrease

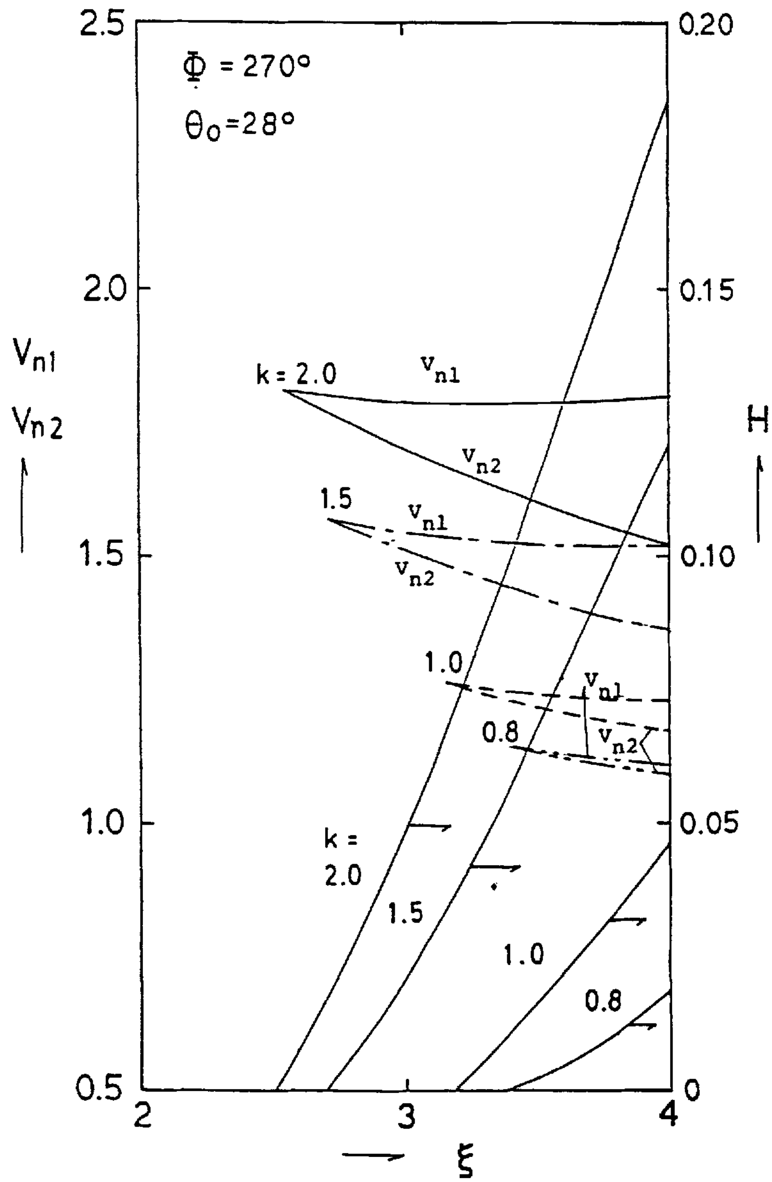

FIGURE 5 Variation of V_{n1}, V_{n2} and H with ξ for k as a parameter. $\theta_o = 28°$, $\Phi = 270°$, $\epsilon_n = 3.6$, $\epsilon_p = 9.9$ and $d = (3/4)P_o$.

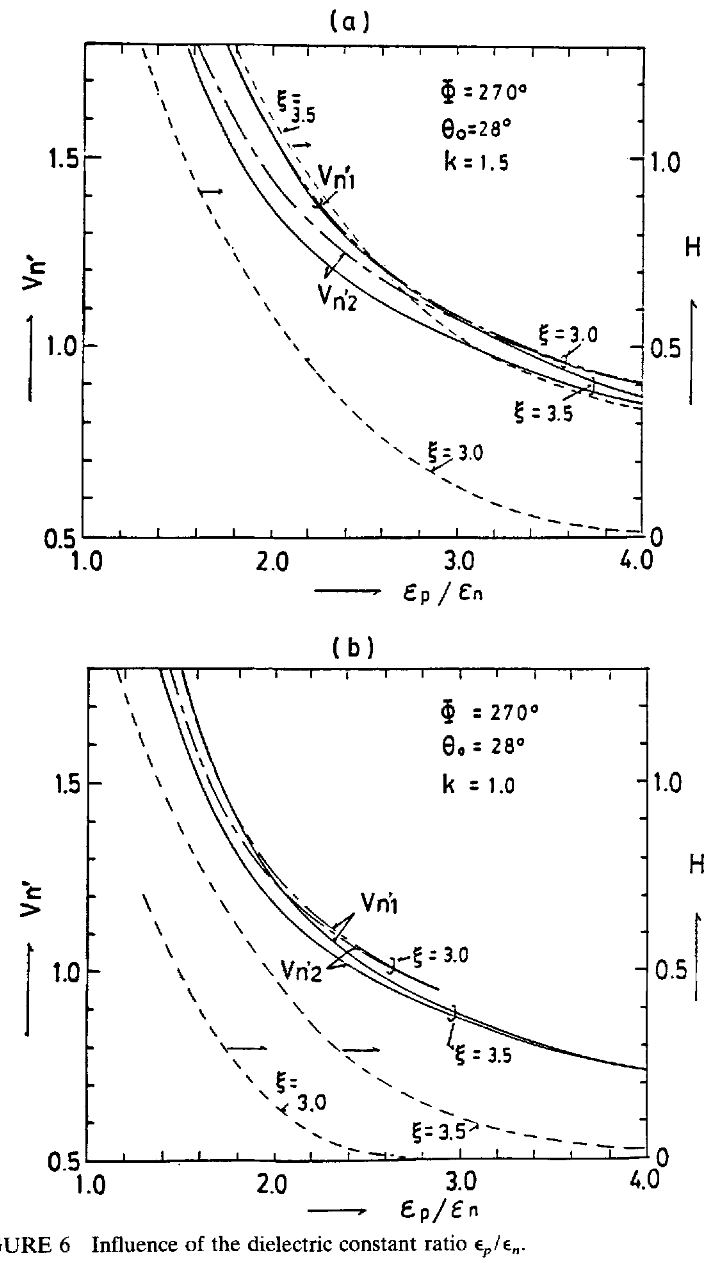

FIGURE 6 Influence of the dielectric constant ratio ϵ_p/ϵ_n.

monotonically with an increase of the dielectric constant ratio ϵ_p/ϵ_n, resulting in a monotonical decrease of H. It should be noted that there exists a critical value of ϵ_p/ϵ_n above which the bistable range completely disappears, and that the critical value of ϵ_p/ϵ_n tends to decrease when k and ξ are decreased.

3.3. Influence of pretilt angle θ_o

The pretilt angle θ_o dependence of θ_m vs the V_n relation is shown in Figure 7. The values of k and ξ are fixed at 1.0 and 3.0, respectively. Other parameters are the same as those in Figure 4.

Note that when θ_o is increased, the bistable range monotonically decreases until it completely disappears.

Transition voltages V_{n1} and V_{n2} and a reduced hysteresis width H are plotted in Figure 8, as a function of θ_o. In this figure, (a) shows the case of $k = 1.0$ and $\xi = 3.0$ and (b) gives the case of $k = 1.5$ and $\xi = 3.5$. From these results, it is found that the critical pretilt angle θ_{oc}, below which the bistability disappears, depends strongly on k and ξ. The critical angle θ_{oc} is plotted for various k and ξ in Figure 9. From these results, it is found that when k and ξ are decreased, a sufficiently high pretilt angle is required to cause the transition between the bistable states. Both transition voltages V_{n1} and

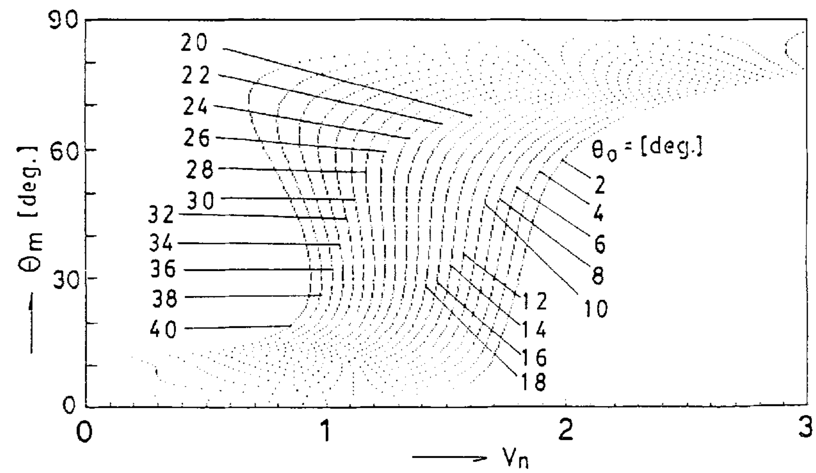

FIGURE 7 θ_m vs V_n with θ_o as a parameter. $\Phi = 270°$, $k = 1.0$, $\xi = 3.5$, $\epsilon_n = 3.6$, and $\epsilon_p = 9.9$.

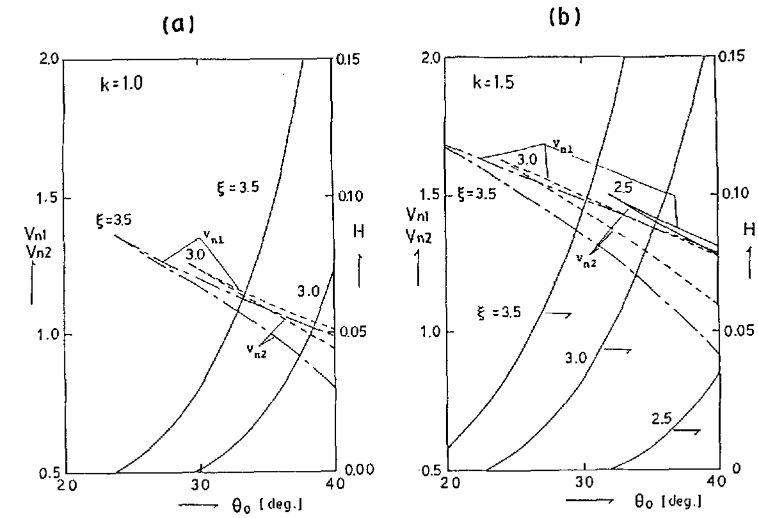

FIGURE 8 Variation of V_{n1} and V_{n2} and H with θ_o for ξ as a parameter. $\Phi = 270°$, $d = (3/4)P_o$, (a) $k = 1.0$, (b) $k = 1.5$.

V_{n2} decrease with an increase of θ_o. On the other hand, a reduced hysteresis width H increases with an increase of θ_o.

3.4. Influence of cell thickness *d*

The variation of θ_m vs V_n for matching factor δ as a parameter is shown in Figure 10. Matching factor δ is defined as

$$\delta = \left(d - \frac{\Phi}{2\pi}P_o\right) \Big/ \left(\frac{\Phi}{2\pi}P_o\right) \tag{21}$$

When no voltage is applied, it may be thought that a conformation of molecular orientation with a total twist angle of 270° can be realized for the range of δ from $-1/3$ to $1/3$.

In the calculation of Figure 10, the values of k, ξ and θ_o were fixed at 1.0, 3.5 and 28°, respectively. When d or δ is increased, both transition voltages V_{n1} and V_{n2} tend to increase and the difference between them decreases until it becomes zero. Thus, with the increase of d, the reduced hysteresis width H monotonically decreases, as shown in Figure 11.

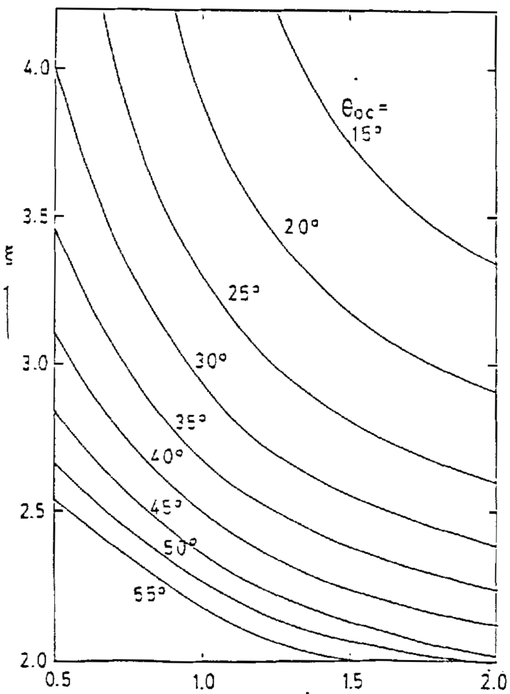

FIGURE 9 Variation of critical pretilt angle θ_{oc} as a function of k and ξ. $\Phi = 270°$, $d = (3/4)P_o$, $\epsilon_n = 3.6$, and $\epsilon_p = 9.9$.

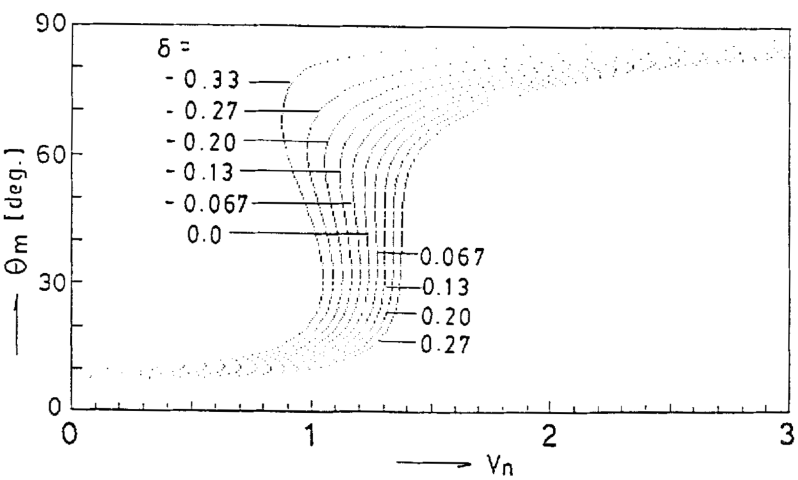

FIGURE 10 θ_m vs V_n with δ as a parameter. $\Phi = 270°$, $\theta_o = 28°$, $k = 1.0$ and $\xi = 3.5$.

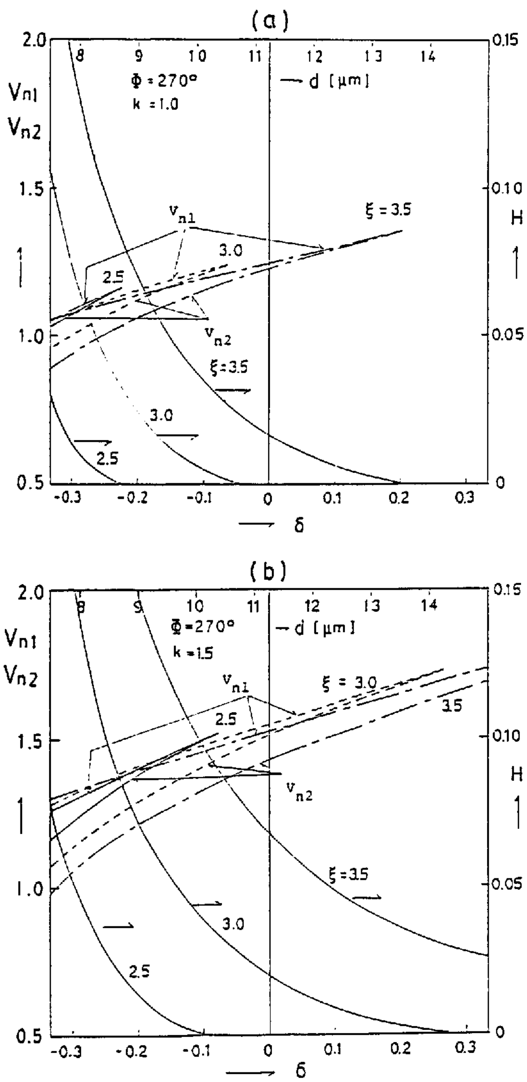

FIGURE 11 Variation of V_{n1}, V_{n2} and H with δ for ξ as a parameter. $\Phi = 270°$, $\theta_o = 28°$, $\epsilon_n = 3.6$ and $\epsilon_p = 9.9$. (a) $k = 1.0$, (b) $k = 1.5$.

The critical value of d or δ, denoted by d_c or δ_c, above which the bistable range disappears, depends strongly on k, ξ and θ_o. The values of δ_c are plotted in Figure 12, as a function of θ_o. The curve parameters are k and ξ. It should be noted that in the case of a low pretilt angle,

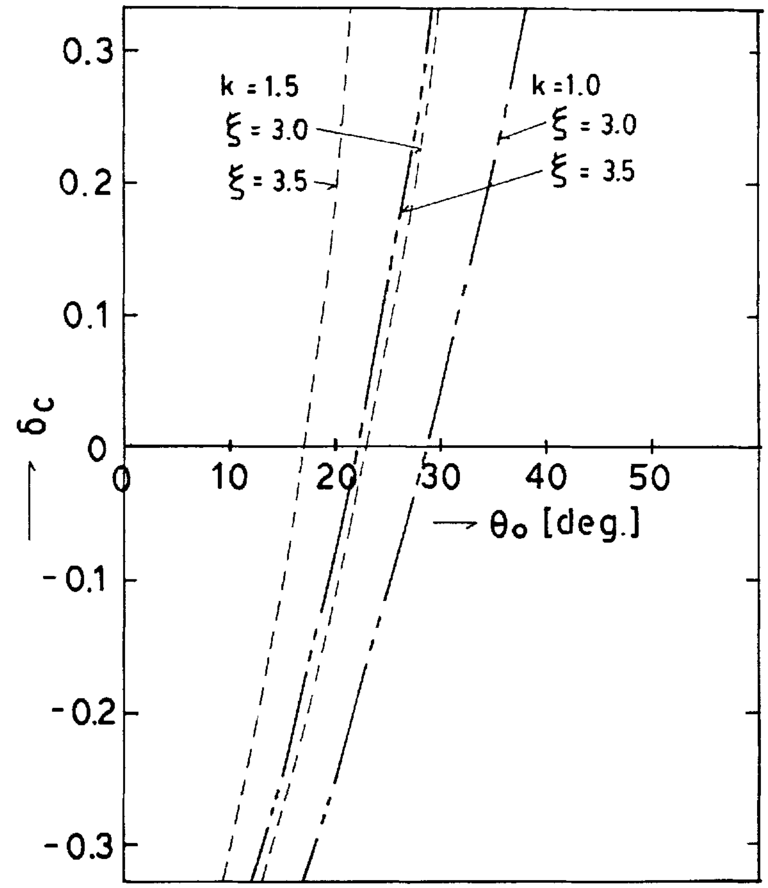

FIGURE 12 Variation of the critical matching factor δ_c with θ_o, for parameters k and ξ; $\Phi = 270°$, $\epsilon_n = 3.6$ and $\epsilon_p = 9.9$.

the value of δ_c becomes smaller than 1/3, and that the value of θ_o decreases as ξ, k and θ_o are decreased, as shown in Figure 12.

3.5. Influence of twist angle Φ

When the total twist angle Φ is decreased, the bistable range also decreases until it completely disappears, as shown in Figure 2. The critical angle, below which no bistable range appears, is plotted as a function of the pretilt angle θ_o, in Figure 13, for some elastic constant ratios $k = k_{33}/k_{11}$ and $\xi = k_{33}/k_{22}$. In these calculations, the thickness/pitch ratio, d/p_o, is varied simultaneously with the total twist angle Φ to maintain the relation $d/p_o = \Phi/2\pi$. From these results, the